Delta Robot

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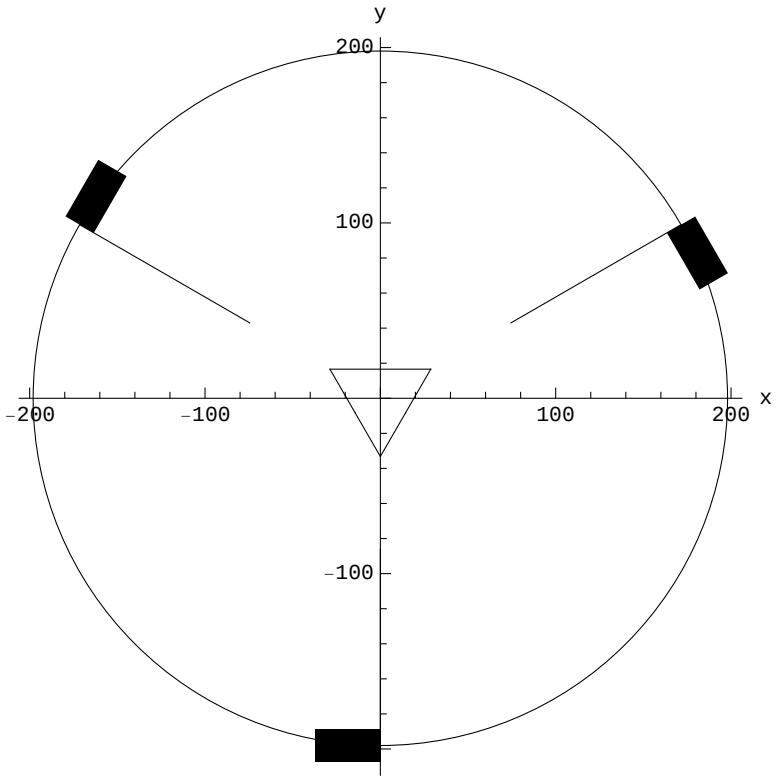
We have the three engines even arranged in a circle with radi = 'engRadiLoc'
and the with an angle of 120 and at the machine coordinate system {ex,ey,ez}.
The whole engine system is turned 'engAngle1' around {ex,ey,ez}
The tool is at {x,y,z}

```
upperRodLength = 112;
lowerRodLength = 232;
f = 457.3;
e = 115 / 2;
engRadiLoc = f / 2 Cos[30 Degree];
toolRadi = e / 2 / Cos[30 Degree];

grTool123 = {
   Line[{{0, 0}, {e / 4, e / 2 Cos[30 Degree]}}],
   Line[{{0, 0}, {-e / 4, e / 2 Cos[30 Degree]}}]
  };

engineDimensions = {upperRodLength / 3, upperRodLength / 6};

{ex, ey, ez} = {1.2 * upperRodLength, 0.9 * upperRodLength, upperRodLength};

grEngine = {
   Rectangle[engineDimensions * {-1, -.5}, engineDimensions * {0, .5}],
   Line[{{0, 0}, {0, upperRodLength}}],
   Translate[grTool123, {0, + (engRadiLoc - 1 * toolRadi)}]};

engAngle1 = 20 Degree;

grEngine1 = Translate[grEngine, {0, -engRadiLoc}];
grEngine123 = Translate[
   Rotate[
    {grEngine1,
     Rotate[grEngine1, 120 Degree, {0, 0}],
     Rotate[grEngine1, -120 Degree, {0, 0}],
     Circle[{0, 0}, engRadiLoc]
    },
    engAngle1, {0, 0}],
   {ex, ey}];
```

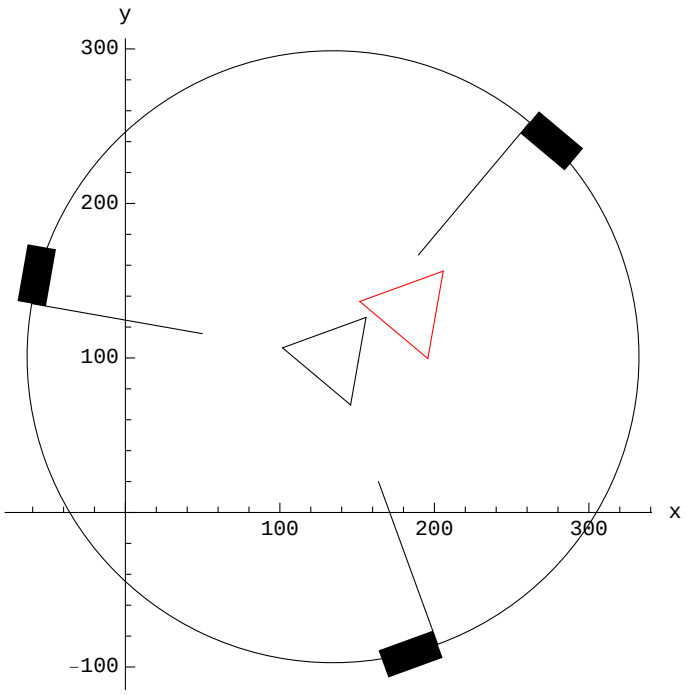
Assume the tool is {50,30,nnn} offset from 3-engine origo

```
desiredOffset = {50, 30};
grTool = {Line[{{-e / 2, e / 2 Tan[30 Degree]}, {e / 2, e / 2 Tan[30 Degree]},
      {0, -e / 2 / Cos[30 Degree]}, {-e / 2, e / 2 Tan[30 Degree]}}]};
grTool = Rotate[grTool, engAngle1, {0, 0}];
grTool = Translate[grTool, {ex, ey} + desiredOffset];
(* Translate {50,30,nnn} in 3 dimensions *)
```

```
Show[
 Graphics
  [{{grEngine1,
     Rotate[grEngine1, 120 Degree, {0, 0}],
    Rotate[grEngine1, -120 Degree, {0, 0}],
    Circle[{0, 0}, engRadiLoc]
   }}
  , Axes -> True, AxesOrigin -> {0, 0}, AxesLabel -> {"x", "y"}
 ]
]
```



```
Show[
 Graphics
  [{grEngine123, Red, grTool}
  , Axes -> True, AxesOrigin -> {0, 0}, AxesLabel -> {"x", "y"}
  ]
]
```



Now calculate for each engine.

Rotate real {x,y,z} around 3-engines origo {ex, ey, ez}

-engAngle1

-engAngle1-120 Degree

-engAngle1=120 Degree

and also transform it with {ex, ey, ez} to get the engine axis at {0,0,0}

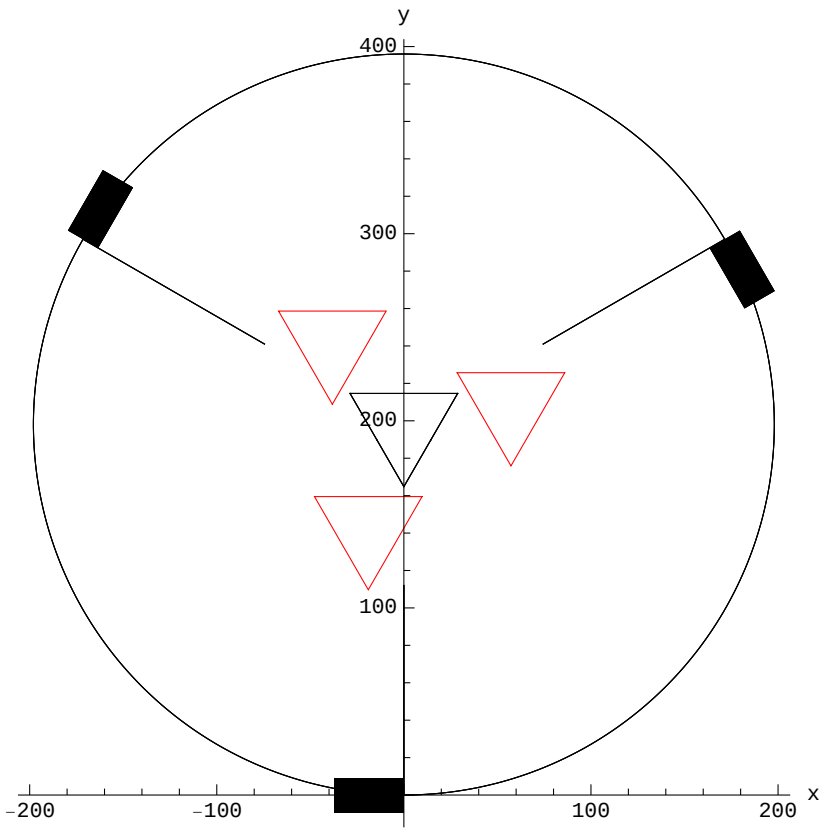
Like:

```
{x, y, z} = fromDesiredSource[xs, ys, zs];
{x0, y0, z0} = rotate[] /. {thePoint -> {x, y, z}, aroundPoint -> {ex, ey, ez}, anAngle
-> -engAngle1};
{x1, y1, z1} = translate[] /. offset -> {- ex, - ey, - ez} + {0, engRadiLoc, 0};

foreach ({0, +120 degree, -120 degree} as toRotate)
{
    {x2, y2, z2} = rotate[] /. {thePoint -> {x1, y1, z1}, aroundPoint -> {0,
+engRadiLoc, ez}, anAngle -> toRotate};
    {x2, y2, z2} -= {0,toolRadi,0} // got middle of lower spherical joints
    {x3, y3, z3} = {0, y2, z2} //  in the y-z-plane
    radiSnittCircle = Sqrt[lenLowerRod^2 - x2^2];
    {{py11, pz11}, {py12, pz12}} = solZeroFunction[radiUpperRod, y3, z3,
radiSnittCircle];
```

```
    choose ArcTan[ {py11, pz11} or {py12, pz12}] as the angle for this engine
}
```

One calcluation might look like

```
Show[
 Graphics
  [{
   grObject = {grEngine123, Red, grTool} // N;
   xxx0 = Rotate[grObject, -engAngle1, {ex, ey}] // N;
   xxx1 = Translate[xxx0, {-ex, -ey + engRadiLoc}] // N;

   xxx2 = Rotate[xxx1, -120 Degree, {0, +engRadiLoc}] // N;
   xxx3 = Rotate[xxx1, +120 Degree, {0, +engRadiLoc}] // N;
   xxx1, xxx2, xxx3
  }
  , Axes -> True, AxesOrigin -> {0, 0}, AxesLabel -> {"x", "y"}
 ]
]
```



Solutions to circle-circle intersection

```
Needs["Optimize`", "/Users/willy/Matte/Optimize-1.m"];
```

SOL two

http://local.wasp.uwa.edu.au/~pbourke/geometry/2circle/

We have
1. a^2 + h^2 == r0^2
2. b^2 + h^2 == r1^2
3. d == a + b
From 2. we get
4. h^2 == r1^2-b^2
Substitute h^2 in 1
5. a^2 + r1^2-b^2 == r0^2
Replacing b in 5 with d - a from 3 gives
a^2 + r1^2-(d-a)^2 == r0^2
Expand this and you get
6. 2 a d-d^2-r0^2+r1^2==0
and then
Calculations:

a = (d^2+r0^2-r1^2)/(2 d) (from 6.)

h = Sqrt[r0^2-a^2] (from 1.)

P2 = P0 + a (P1 - P0) / d
x3 = x2 +- h (y1 - y0) / d
y3 = y2 -+ h (x1 - x0) / d

This is written as a function

```
interSect2Circ[{x0_, y0_}, r0_, {x1_, y1_}, r1_] :=
 Module[{d, a, h, p2x, p2y, x31x, y31y, x32x, y32y},
  d = Sqrt[(x1 - x0)^2 + (y1 - y0)^2];
  If[d > r0 + r1, Return[{d, "Separated", {0, 0}, {0, 0}}]];
  If[d < Abs[r0 - r1], Return[{d, "Contained", {0, 0}, {0, 0}}]];
  If[d == 0 && r0 == r1, Return[{d, "Same", {0, 0}, {0, 0}}]];
  a = (r0^2 - r1^2 + d^2) / (2 d);
  {p2x, p2y} = {x0, y0} + a ({x1, y1} - {x0, y0}) / d;
  h = Sqrt[r0^2 - a^2];
  {x31x, y31y} = {p2x + h (y1 - y0) / d, p2y - h (x1 - x0) / d};
  {x32x, y32y} = {p2x - h (y1 - y0) / d, p2y + h (x1 - x0) / d};
  {d, {x31x, y31y}, {x32x, y32y}}
 ]
```

```
solTwo = interSect2Circ[{0, 0}, r1, {y, z}, r2]
```

$$\left\{\sqrt{y^2+z^2},\ \left\{\frac{y\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}+\frac{z\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}},\right.$$

$$\left.\frac{z\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}-\frac{y\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}}\right\},$$

$$\left\{\frac{y\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}-\frac{z\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}},\right.$$

$$\left.\frac{z\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}+\frac{y\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}}\right\}\right\}$$

A source modifivation of two for zero location

```
interSect2CircZero[r0_, {x1_, y1_}, r1_] :=
 Module[{d, a, h, p2x, p2y, x31x, y31y, x32x, y32y},
  d = Sqrt[x1^2 + y1^2];
  If[d > r0 + r1, Return[{d, "Separated", {0, 0}, {0, 0}}]];
  If[d < Abs[r0 - r1], Return[{d, "Contained", {0, 0}, {0, 0}}]];
  If[d == 0 && r0 == r1, Return[{d, "Same", {0, 0}, {0, 0}}]];
  a = (r0^2 - r1^2 + d^2) / (2 d);
  {p2x, p2y} = a * {x1, y1} / d;
  h = Sqrt[r0^2 - a^2];
  {x31x, y31y} = {p2x + h * y1 / d, p2y - h * x1 / d};
  {x32x, y32y} = {p2x - h * y1 / d, p2y + h * x1 / d};
  {{x31x, y31y}, {x32x, y32y}}
 ]
```

```
solTwo = interSect2CircZero[r1, {y, z}, r2]
```

$$\left\{\left\{\frac{y\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}+\frac{z\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}},\right.\right.$$

$$\left.\frac{z\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}-\frac{y\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}}\right\},$$

$$\left\{\frac{y\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}-\frac{z\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}},\right.$$

$$\left.\left.\frac{z\left(r1^2-r2^2+y^2+z^2\right)}{2\left(y^2+z^2\right)}+\frac{y\sqrt{r1^2-\frac{\left(r1^2-r2^2+y^2+z^2\right)^2}{4\left(y^2+z^2\right)}}}{\sqrt{y^2+z^2}}\right\}\right\}$$